The stochastic scenario method for modeling uncertainty in computable equilibrium models

Application to European energy markets

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Introduction

- Basis for the project: LIBEMOD, a CGE model of the European energy market developed over long time.
- Extension: Agents simultaneously maximizes facing uncertainty.
- Uses an approach of scenario aggregation.
- Add two types of uncertainty:
  - Uncertainty about GDP-growth and oil prices.
  - Uncertainty about future climate policy.

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- Adding this side constraint modifies the first order conditions
  - Only a moderate change in the model.

The math

- Chose optimal x when ξ is stochastic; $\xi = \xi_s$ with probability p_s:

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$$\max_{x_1, x_2, ..} \left[\sum_{s \in S} f(x_s, \xi_s) p_s \right]$$

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- Note similarity to a set of deterministic problems (the existing model)

$$\max_{x_s} \left[f(x_s, \xi_s) p_s \right] \text{ for a given } s$$

but with the constraint $x_s = x_n$ for all $s \in S, s \neq n$ linking the problems.

The Lagrange function

- The problem

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- Give a Lagrange function of the form

$$\begin{aligned} L &= \sum_{s \in S} f(x_s, \xi_s) p_s + \sum_{s=1}^{n-1} \lambda_s (x_s - x_n) \\ &= \sum_{s \in S} [f(x_s, \xi_s) p_s + \lambda_s x_s] = \sum_{s \in S} L_s \end{aligned}$$

with

$$\lambda_n = -\sum_{s \neq n} \lambda_s$$

First order conditions

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- After normalization

$$\tilde{\lambda}_s = \frac{\lambda_s}{p_s}$$

the conditions are

The original model	**Scenario aggregation**	**Comments**
$\nabla_{x_s} f(x_s, \xi_s) = 0$	$\nabla_{x_s} f(x_s, \xi_s) + \tilde{\lambda}_s = 0$	Add shadow price for $x_s = x_n$
	$E\tilde{\lambda} = 0$	Constraint on shadow prices
	$x_s = x_n$ for all $s \neq n$	Policy must be implementable

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$$\text{Capacity each region} = 5.5 = \frac{1+10}{2}$$
$$\text{Transmission capacity} = 0$$

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- Optimal solution: not the average, but something different. We seek flexibility.

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- For the current application simpler appraches sufficient

Uncertainty about GDP and oil prices.

- Uses observations from 1970 to 2010,
- We think of this as four 10-years period.
- The future developmen from 2010 to 2030 is two 10-years periods.
- Each future 10-years period may be like one in the past with respect to GDP growth and relative changes in oil price.
- Makes 16 different scenarios, but only 10 unique.

Results GDP/Oil uncertainty in LIBEMOD.

	Deterministic	Monte Carlo	Sotchastic
New capacity, GW	365	354	358
Hydro	9	10	11
Bio	13	15	17
Wind	9	28	31
Oil	0	13	0
Gas	30	37	49
Coal	304	250	250
NewTransm, el. GW	5	19	16
NewTransm. Gas, MTOE	157	157	154

Results - continued.

- In general more green technologies under uncertainty.
  - A coal plan may end up not being used in marginal costs are too high
  - The model predicts more than 3 times as much wind under uncertainty than with certainty to replace coal.
- Monte Carlo (average of the 10 scenarios) similar to stochastic, but
  - Predicts investment in oil-plants that are tailor suited to in only one scenario but not optimal under uncertainty.

Results in political uncertainty in LIBEMOD - Investments.

Two scenarios: With/without a 44 USD/TCO_2 tax.

	Deter.	Stoch.	M. Carlo	No Tax	Tax
New cap. GW	336	360	365	365	365
Hydro	14	14	13	9	17
Bio	25	25	23	13	33
Wind	111	105	110	9	210
Oil	0	0	0	0	0
Gas	66	72	67	30	105
Coal	120	144	152	304	0
New Transm, elect. GW	19	20	30	5	55
NewTransm. , Gas, MTOE	146	136	165	157	174

- Uncertainty pretty much like a 22 USD/TCO_2 Tax
- A 44 USD/TCO_2 tax would have much stronger effect if it was credibly announced.

Results in political uncertainty in LIBEMOD - Production.

Two scenarios: With/without a 44 USD/TCO_2 tax.

	Determ	Stoch_No	Stoch_Tax	MC_No	MC_Tax
Tot. prod. TWh	4126	4346	3644	4851	3467
Hydro	475	474	474	461	484
Renewable	626	605	609	226	959
Fossil	2189	2430	1725	3329	1188
Nuclear	836	836	836	836	836

- An uncertain tax of 22 USD/TCO_2 that only lingers until 2030 has a stronger effect than a 22 USD/TCO_2 tax
- If announced in 2010 fossil fuel cut 64%, if finanlized only in 2030 the effect is 48% compared to 32% for a 22 USD/TCO_2 tax.

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- The results change only modestly with modest uncertainty
- Substantial effects with much uncertainty
- Lots of sources of uncertainty still left out – for future work.

Further extensions, Dynamic models

- Two periods, four scenarios

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\begin{aligned}
S_1 &= \{1, 2, 3, 4\} \ (t = 1) \\
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  - Current irreversibility: $K_{14} \geq K_0$
  - Future irreversibility: $K_{22} \geq K_{14}$ and $K_{24} \geq K_{14}$
- Expected value of shadow prices no longer zero

Dynamic models (continued)

- General first-order conditions (complementary slackness):

$$
\begin{aligned}
p_{ts} - \frac{rc}{1+r} + \tilde{\lambda}_{ts} &= 0 \\
(\tilde{\lambda}_{ti} - E\tilde{\lambda}_{t+1,i})(K_{ts} - K_{t-1,s}) &= 0 \quad \text{for } s \in S_{it} \\
E\tilde{\lambda}_{t+1,i} &= \sum_{j} q_j \tilde{\lambda}_{t+1,l}
\end{aligned}
$$

$$
(\text{summing over } l \;\; \text{s.t. } S_{t+1,l} \subset S_{ti})
$$

and

$$
\tilde{\lambda}_{ti} = E_i\left(\tilde{\lambda}_{ts}\right) = \frac{\sum_{s \in S_{ti}} q_s \tilde{\lambda}_{ts}}{\sum_{s \in S_{ti}} q_s}
$$

$$
q_j = \frac{\sum_{s \in S_{tj}} q_s}{\sum_{s \in S_{ti}} q_s}.
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