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A model for supply of informal care to elderly parents[1]

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Abstract

This paper presents a model of informal labour supply to parents. We assume that the child participates in the labour market and gains in utility from consumption and leisure. In addition it has altruistic motivation to give informal care to its elderly parent. We show how the labour income, labour supply and informal caregiving are affected by exogenous factors such as the education level, wage rate, other supply of care, travel distance and inheritance.

1. Introduction

The theoretical literature emphasises several motives for a grown up child to provide care to its elderly parents (see, e.g., Giménez et al., 2007). Some examples are altruism, duty, social norms, reciprocity, exchange (money transfers as payment for services provided by children), loan transfers, strategic bequest motives and a demonstration effect (the care giver wants to affect the habits or norms of their children as a mean to receive future care). The bulk of the literature, however, focuses on altruism and exchange. In an altruistic model, the child takes the utility function of the parent into account when determining its behaviour. Some examples of this are found in Chang and White-Means (1995), Nocera and Zweifel (1996) and Kuhn and Nuscheler (2007). In this paper we assume altruism to be the motivation behind caregiving.

The paper is meant as a companion paper to our analysis in Fevang et al. (2008), where we study empirically the effects on grown up children's income of giving informal care

[1] We are indebted to Jos van Ommeren for discussions and comments.

to an elderly parent. The model is formulated in a flexible way to be able to analyse many of the factors that have an effect of labour market outcomes and that are studied in the empirical analysis. Also, the comparative statics provided below is based on the variables available in the data.

2. The basic model

We consider a model for supply of informal care, where the recipient of informal care is not in the household of the caregiver. Thus, the disability of the recipient will not directly affect household income or household production of the caregiver (see, e.g., Ahlburg and Chi, 2006, for a model on supply of informal care within the household). We name the caregiver as *child* and the recipient as *parent*.

The parent is in need of care in the period before death, which is partly provided by the child (informal care) as well as by others. The latter can be informal care, public care or market care. We assume that other care is independent on the caregiving of the child. This can be thought of as the parent has only one child[2] (as in Kuhn and Nuscheler, 2007) or that other care is only formal care (public or private).

We focus on three periods where the initial period ($t=0$) is the period when the parent is healthy and not in need of any care. Period 1 ($t=1$) is the period before the death of the parent, and where she is in need of care. The final period ($t=2$) is the period after death, and the child provides no caregiving.

The utility function of the child at time $t=1$ is given by:

(1) $U_1 = v(C_1) + \beta V(N)$

[2] We could assume that other care is care provided by siblings. However, there may be a game between siblings as one child can free ride on the care provided by other children. Several papers have studied the interactions or strategic behaviour of siblings when the parent's health is considered a common good, see, e.g., Konrad et al. (2002), Engers and Stern (2002), Rainer and Siedler (2005) and Callegaro and Pasini (2007). Also, empirical studies find that a child provides less care to its parent when it has siblings, see, e.g., Romøren (2003).

Where v is a concave function in total consumption denoted as C. As in Kuhn and Nuscheler (2007), we assume that the child cares about the wellbeing of its parent, and that this is presented as an additive altruistic term where β in general lies between 0 and 1. V is the utility function of the parent, which is concave in the total amount of care denoted by N.

At time $t=0$ and $t=2$, the parent is either not in need of care or has deceased, and the utility function of the child is dependent on total consumption only:

(2) $U_t = v(C_t),\ t=0,2$

Total consumption is assumed to be a household production function of consumed goods, X, and leisure, L, where $C'_X > 0,\ C''_{XX} < 0,\ C'_L > 0,\ C''_{LL} < 0$. In addition, we follow the standard human capital approach and assume that education, E, changes the productivity of time and goods positively in producing total consumption (see, e.g., Michael, 1973), i.e., $C''_{XE} > 0$ and $C''_{LE} > 0$:

(3) $C_t = C(X_t, L_t; E)$

As a simplification we assume that the utility of consumption can be written in the following way:

(4) $u(X_t, L_t; E) = v(C(X_t, L_t; E))$

We then find $u_X > 0,\ u_{XX} < 0,\ u_L > 0$ *and* $u_{LL} < 0.$ We assume $u_{XE} > 0,\ u_{LE} > 0,\ u_{XL} > 0.$[3]

Thus, we can rewrite equations (1) and (2) as follows:

(5) $U_0 = u(X_0, L_0; E)$

[3] We find $u_X = v_C C_X > 0$, $u_{XX} = v_{CC} C_X + v_C C_{XX} < 0$, $u_{XE} = v_{CC} C_E C_X + v_C C_{XE}$, $u_L = v_L C_L > 0$, $u_{LL} = v_{LL} C_L + v_L C_{LL} < 0$, $u_{LE} = v_{CC} C_E C_L + v_C C_{LE}$ and $u_{XL} = v_{CC} C_X C_L + v_C C_{XL}$.

(6) $U_1 = u(X_1,L_1;E) + \beta V(N)$

(7) $U_2 = u(X_2,L_2;E)$

The level of care received by the parent is the sum of informal care supplied by the child, Z, and other care, $\overline{Z}$:[4]

(8) $N = Z + \overline{Z}$

Finally, the income, Y, in the three periods can be written as follows:

(9) $Y_0 = w(T_0 – L_0)$

(10) $Y_1 = w(T_1 - L_1 – (1+a)Z)$

(11) $Y_2 = w(T_2 - L_2) + M$

T_t is the total time available for work, leisure and caregiving at time t. This can be thought of as healthy time, i.e., time not being sick. a is a travel cost related to giving care,[5] w is the wage rate and M is the inheritance the child receives after the death of the parent.

We will now consider two versions of the model. In the first version, the child is rationed in the credit market, while in the second version there is a perfect credit market.

3. An imperfect credit market

Assume that the credit market is imperfect so that the child is not able to transfer money from one period to another. Thus we have the following budget constraints for the three

[4] We assume that informal care is a perfect substitute to public or market care. In contrast, Kuhn and Nuscheler (2007) analyse the optimal public provision of nursing homes, and assumes that there may be a productivity difference between nursing homes and family care. In addition, they assume a fixed utility loss for parents when moving to nursing homes.

[5] Disregarding the time to travel, one unit of time is transferred into one unit of informal care. Thus, we assume that the productivity of informal care production is independent of the level of education of the caregiver. This is supported by Norwegian data (see, e.g., Romøren 2003; Gautun, 2003).

periods, where P is the price of the consumption good and c is the monetary cost of providing care which may represent travel distance:[6]

(12) $PX_0 = Y_0$

(13) $PX_1 + cZ = Y_1$

(14) $PX_2 = Y_2$

Period 0

The optimisation problem of the child in period 0 is to maximise equation (5) with respect to X_0, L_0 given (9) and (12).

The first order conditions from this optimisation problem can be written as follows:

(15) $\frac{u'_X}{u'_L} = \frac{P}{w}$

As seen, the marginal rate of substitution (MRS) between consumption and leisure should equal the relative price between consumption good and time. Together with the budget condition ((9) and (12)), this determines the optimal values of X_0 and L_0.

Period 1

The optimisation problem of the child in period 1 is to maximise equation (6) with respect to X_1, L_1 and Z given (8), (10) and (13).

The first order conditions from this optimisation problem can be written as follows:

(16) $\frac{u'_X}{u'_L} = \frac{P}{w}$

[6] If c=0, X_1 is a proxy of labour supply and income as all prices (P, w and a) are constant. In this case, $U''_{LX} > 0$ also means that the productivity of leisure will increase in income, which is a sufficient condition for leisure to be a normal good.

$$\text{(17)} \quad \frac{u_X^{'}}{\beta V_N^{'}} = \frac{P}{w(1+a)+c}$$

Thus, the marginal rate of substitution (MRS) between consumption and leisure as well as consumption and caregiving should equal the relative price between these goods. While the price of leisure is a pure time cost, the price of care consists of both a time cost and a monetary cost.

From equations (16) and (17), we also find:

$$\text{(18)} \quad \frac{u_L^{'}}{\beta V_N^{'}} = \frac{w}{w(1+a)+c}$$

This means that the marginal rate of substitution between leisure and caregiving should equal the relative difference in costs between these two alternatives.

Equations (16), (17) and the budget condition given by (10) and (13) determine the optimal levels of consumption goods (X_1), leisure (L_1) and caregiving (Z) in period 1. Denote the optimal levels as X_1^*, L_1^* and Z^*. We then find

(19) $X_1^* = X_1^*(\beta, E, \overline{Z}, T_1, w, P, a, c)$

(20) $L_1^* = L_1^*(\beta, E, \overline{Z}, T_1, w, P, a, c)$

(21) $Z^* = Z^*(\beta, E, \overline{Z}, T_1, w, P, a, c)$

We will now study how changes in the education level (E), the wage rate (w), other caregiving ($\overline{Z}$) and travel costs (c and a) affect the optimal levels of these variables.[7]

[7] See Appendix B for more details.

A. A higher education level

We first study an increase in the education level (E) for a constant wage rate. Here we do not consider a direct income effect from education, but we are able to study different behaviour of people with the same wage rate but different education level, i.e., the partial effect of education on caregiving.

As $u''_{XE} > 0$ and $u''_{LE} > 0$, the marginal benefits from an increase in consuming the consumption good as well as leisure will increase. This gives two contradictory effects: First, this increases the marginal utility of consumption and leisure compared to caregiving.[8] This goes in the direction of less caregiving and more consumption and leisure. However, there is a second contradictory effect. As the child gets a higher utility for a given amount of consumption or leisure, she does not need the same level of input of X and L for a given utility level. This goes in the direction of less purchase of the consumption good and less leisure. As a result, caregiving may actually increase.

However, as seen from equations (16)-(18), as the child becomes more productive in producing the household consumption good, the MRS between the consumption good and caregiving (equation (17)) as well as the MRS between leisure and caregiving (equation (18)) have increased. Thus caregiving has to go down to restore the equilibrium. Under reasonable conditions,[9] the extra time (and money) from a reduction of caregiving will be spent on both working (and consumption) and leisure.

Result 1: *Caregiving is lower the higher the education level is. Under reasonable conditions, income will increase in the education level.*

[8] Alternatively, this can be interpreted as the child becomes more productive in producing total consumption, see equation (3), than caregiving; the price of producing one unit of consumption goes down relative to the price of producing one unit of care

[9] This means that all extra time will not go to only one of the alternatives, which is reasonable as the marginal benefits of the different alternatives are falling. However, as the monetary cost of caregiving, cZ, goes down as Z falls, consumption will increase even if labour supply is constant. Hence, there is a possibility that labour supply does not increase.

B. A higher wage rate

An increase in the wage rate can also be thought of as an effect of education, but a different effect than the one studied above. In the empirical analysis (Fevang et al., 2008), we study the effects of parent's income. Assuming that own income is correlated to parent's income, it may serve as a proxy for parent's income. Further, if there is a gender difference in wage rate, this analysis may also explain parts of the gender difference.

A higher wage increases the budget. This gives standard contradictory effects; the income and substitution effects. First, as the wage rate goes up, the child does not have to work as much as before to earn the same income, thus more time is available for leisure and caregiving. On the other hand, the alternative cost of leisure and caregiving increases, which goes in the direction of higher labour supply. We find that the effects on income and caregiving are indeterminate and we cannot tell which effect dominates.

Result 2: *We cannot in general tell the effect on income and caregiving of a change in the wage rate.*

C. Higher exogenous caregiving

An increase in $\overline{Z}$ can be interpreted as a higher supply of informal[10] as well as formal or public care, but also as a change in the municipal organisation of care supply from home care services to institutions as this may increase the total amount of care given to the parent.

In general, higher public contributions may crowd out private contributions, see, e.g., Nyborg and Rege (2003). In our model, a higher $\overline{Z}$ will increase the total amount of care received by the parent (*N*), everything else given. This will reduce the marginal utility of the parent from a unit of care; $V'(N)$. Thus, the MRS between consumption and caregiving as well as between leisure and caregiving will increase; see equations (17) and

[10] This can be interpreted as more siblings, but only under strict assumptions as more siblings does not necessary mean an increase in total caregiving, see footnote 2.

(18). As a result, under reasonable conditions,[11] the child will increase its purchase of consumption goods, increase labour supply and leisure and reduce its caregiving.

Result 3: *An increase in public care will reduce informal caregiving and, under reasonable assumptions, increase the labour supply and income of the caregiver.*

D. Higher travel costs

Travel costs can either be time costs or monetary costs. Small (1992) finds that about two thirds of the costs of commuting are time costs, while only one third is a monetary cost. Time costs and monetary costs can have different effect on labour supply. The study by Cogan (1981) shows the effects of fixed costs associated with entry into the labour market. An increase in a monetary fixed cost will increase labour supply for those who continue to work, while an increase in the time cost will reduce hours of work among workers.

A high a and/or c in our model can be interpreted as a longer geographical distance from the child to the parent, and thus higher travel costs. Note that this can be a further implication of education in addition to the analysis above, as children with higher education usually move further away from home compared to children with lower education.

We first consider a higher monetary cost. Assume that there is an increase in c. Higher travel costs increases the price of providing informal care and the supply of informal care goes down. Further, under reasonable conditions, we find that leisure time, consumption and labour supply will increase.[12]

However, if the marginal utility of caregiving is sufficiently high, an increase in travel costs may actually lead to lower leisure. The reason is that more labour should be

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[11] See footnote 9.

[12] Again, if the total monetary cost of caregiving, cZ, goes down, consumption will increase even if labour supply is constant, and there is a possibility that labour supply does not increase. But note that cZ does not necessary go down in this case as c increases.

supplied to be able to provide care (to pay for the travel costs), and leisure has to be reduced.

Result 4: *Higher monetary travel costs will reduce informal caregiving and, under reasonable conditions, increase leisure and labour supply.*

Assume instead that travel costs are time costs. A rise in the time cost implies that more time is required to provide a given level of care, which gives a reduction in labour supply and leisure. But, this also increases the price on informal care and leads to lower care provision. However, even if the level of care provision is lower, the total time spent on care may be higher, and we cannot in general tell whether labour supply and leisure will increase.

Result 5: *Higher time costs of care giving will reduce the level of informal caregiving but not necessary the time spent on caregiving. Thus, the overall impact on labour supply and leisure is indeterminate.*

Period 2

In period 2, the parent has deceased and the child maximizes the utility function in equation (7), with respect to X_2 and L_2 given (11) and (14).

The first order conditions from this maximization problem is:

$$(22)\quad \frac{u_X^{'}}{u_L^{'}} = \frac{P}{w}$$

Thus, as in period 1, the marginal rate of substitution (MRS) between consumption and leisure should equal the relative price between consumption good and time. Together with the budget constraint, (11) and (14), this determines the optimal allocation of time between work and leisure, and the purchase of the consumption good.

Note that in this period of time, X_2 is not a proxy of income, as the child also has received an inheritance (M). However, for a given inheritance, a change in X_2 is equal to a change in labour income. Also, for a constant available time, T_2, an increase in leisure, L_2, reduces working time.

As the parent has deceased, the child receives an inheritance (M). Receiving an inheritance will have impacts on the optimal levels of work and leisure. We find that the child increases his leisure time.[13] An increase in leisure means a fall in time devoted to work and a fall in labour income.[14]

The intuition is as follows. A positive inheritance will increase the consumption possibilities of the child. This can be used to increase the purchase of consumption goods, increase time used for leisure or both. As time can only be used for work and leisure, devoting more time to leisure means a lower labour supply. Whether the child wants to reduce labour supply or not is dependent on the specification of the utility function. If more leisure and consumption are complementary in utility ($u''_{LX} > 0$), the child wants to take out some of the wealth increase in leisure and reduce its labour supply. Note also that as the credit market is imperfect, the inheritance in period 2 does not affect labour supply in period 1.

Result 6: *With an imperfect credit market, the child will supply less labour in period 2 if there is an inheritance from the parent.*

The loss of a parent may also affect the health of the child in several ways. The child may be exhausted after a long period of nursing, or sorrow may reduce the ability to work. We interpret this as a reduction in healthy time, T_2. As expected, in Appendix B we find that

[13] See Appendix B for comparative statistics.

[14] This is a standard result in the literature as long as leisure is a normal good. Note that $\frac{dL_2}{dM} = \frac{wu''_{XX} - Pu''_{LX}}{u''_{LL}P^2 + w\left(wu''_{XX} - 2Pu''_{LX}\right)}$ and the result depends on $u''_{LX} \geq 0$, which is a sufficient condition for leisure to be a normal good.

consumption goes down for a reduction in T_2, which means that labour income has been reduced.

Result 7: *A reduction in available or healthy time in period 2 will reduce the labour income of the child.*

A comparison of labour supply across periods

Assume equal time budgets across periods $(T_0 = T_1 = T_2)$. We can now compare the labour supply for the different periods. With credit constraints, we normally expect labour supply to decline from period 0 to period 1, as the reduction in available time for leisure and consumption is distributed between the two goods in order keep the marginal rate of substitution constant. However, if the monetary cost of care-giving c is sufficiently high as well as parent's marginal utility of care, labour supply may actually increase in period 1. In this case it would be optimal for the child to reduce leisure and increase labour supply in order to be able to provide the costly care. With credit constraints, the labour supply in period 2 is unequivocally lower than in period 0, since the inheritance entails an income effect raising both consumption and leisure in period 2. Compared to period 1, labour supply may rise or fall, depending on whether the removal of the care requirements or the income effects arising from inheritance dominates.

4. A perfect credit market

As we saw above, an inheritance in period 2 does not affect the labour supply in period 1 in an imperfect credit market. Even with a perfect credit market, inheritance in period 2 may not have much influence on labour supply in period 2 if the size of the inheritance is uncertain. Thus, in this case we may obtain similar conclusions as in the analysis above. However, if there is a perfect credit market and no uncertainty about the size of the inheritance,[15] the child will take this into account when entering the labour market. The

[15] E.g., the size of the inheritance is independent of the time of death.

size of the inheritance will affect labour supply in all periods, but the labour market decision will not be affected by the time of the death of the parent.[16]

As we only have three periods in the model, we can study how the labour supply in period 0 and 1 are affected by an inheritance in period 2 in a perfect credit market, when there is no uncertainty about the size of the inheritance. In the following, we disregard interest rates and discounting,[17] thus the intertemporal budget condition of the child is:

$$(23)\quad pX_0 + pX_1 + cZ + pX_2 = w(T_0 - L_0) + w(T_1 - L_1 - (1+a)Z) + w(T_2 - L_2) + M$$

The child maximises the intertemporal utility function

$$(24)\quad U = \sum_{t=0}^{2} U_t$$

with respect to X_0, X_1, X_2, L_0, L_1, L_2 and Z, given equations (5)-(8) and (23).

The first order conditions from this optimisation problem are given by equations (15), (16), (18), (22), (23) and the intertemporal conditions:

$$(25)\quad \frac{u'_{X_1}}{u'_{X_2}} = 1$$

$$(26)\quad \frac{u'_{X_0}}{u'_{X_2}} = 1$$

We find that labour supply in period 0 and 1 are reduced for an increase of the inheritance. The reason is that the child finds it optimal to spread the consumption of the

[16] As there is a perfect credit market and the size of the inheritance is known, the time path will be time consistent and independent of the time of the death of the parent.
[17] We can easily show that if the market interest rate is set equal to the discount rate, disregarding interest rates will not affect the first order conditions.

inheritance over the three periods, as there are diminishing returns from consumption, and then does not need to work that much to be able to achieve the same consumption level. Consider for instance period 0. As seen from (15), consuming the inheritance for a given labour supply will reduce the MRS between consumption and leisure. In the new equilibrium, the child will therefore increase leisure and reduce labour supply. The intuition will be similar for period 1.

Result 8: *In a perfect credit market with no uncertainty about the size of the inheritance, a higher inheritance will reduce the labour supply in period 0 and 1.*

Assume equal time budgets across periods $(T_0 = T_1 = T_2)$. We can now compare the labour supply for the different periods. In the absence of credit constraints and uncertainty, this model predicts labour supply to be lower in period 1 than in periods 0 and 2 (provided an interior solution to the optimization problem), as total time are spread over three activities in this period. Also, we find labour supply to be equal across the pre- and post-care periods, see (15), (22) and (26).

Doing comparative statics on other exogenous variables such as E and w, would provide similar results found under the imperfect credit market.

5. Possible extensions

This model opens up for several analysis and extensions. One of the interesting aspects would be to study further the effects of inheritance. The model can be extended to include uncertainty for instance about the size of inheritance. We could also study the response to inheritance for, e.g., different levels of education and wage. Another extension is to study the age effect. The age of the child may matter for instance for the time budget (depending for instance on the age of its own children), marginal utilities and credit constraints. Strategic considerations are also not considered in this model. This could be considerations for giving care other than altruism, or strategic interactions among siblings.

Appendix A: The optimisation problem

Note: This appendix is not yet finished.

1. Imperfect credit market

Period 1

The child wants to maximise

(A1) $U_1 = u(X_1, L_1; E) + \beta V(Z + \overline{Z})$

given

(A2) $PX_1 + cZ = w(T_1 - L_1 - (1+a)Z)$

This gives the following Lagrangian:

(A3) $L_{per1} = u(X_1, L_1; E) + \beta V(Z + \overline{Z}) + \lambda_1 (w(T_1 - L_1 - (1+a)Z) - PX_1 - cZ)$

First order conditions are:

(A4) $\dfrac{\partial L_{per1}}{\partial X_1} = u_X^{'} - \lambda_1 P = 0$

(A5) $\dfrac{\partial L_{per1}}{\partial L_1} = u_L^{'} - \lambda_1 w = 0$

(A6) $\dfrac{\partial L_{per1}}{\partial Z} = \beta V_N^{'} - \lambda_1 (w(1+a) + c) = 0$

(A7) $\dfrac{\partial L_{per1}}{\partial \lambda_1} = w(T_1 - L_1 - (1+a)Z) - PX_1 - cZ = 0$

This gives:

(A8) $wu_X^{'} = Pu_L^{'}$

(A9) $(w(1+a) + c)u_L^{'} = w\beta V_N^{'}$

(A10) $w(T_1 - L_1 - (1+a)Z) - PX_1 - cZ = 0$

Equations (A8)-(A10) determine X_1, L_1 and Z.

The second order conditions can be written as follows:

(A11)

(A12)

Note: Second order conditions may be useful to find the signs in the comparative statics below.

<u>Period 2</u>

The child wants to maximise

(A13) $U_2 = u(X_2, L_2; E)$

given

(A14) $PX_2 = w(T_2 - L_2) + M$

This gives the following Lagrangian:

(A15) $L_{per2} = u(X_2, L_2; E) + \lambda_2(w(T_2 - L_2) + M - PX_2)$

First order conditions are:

(A16) $\dfrac{\partial L_{per2}}{\partial X_2} = u_X^{'} - \lambda_2 P = 0$

(A17) $\dfrac{\partial L_{per2}}{\partial L_2} = u_L^{'} - \lambda_2 w = 0$

(A18) $\dfrac{\partial L_{per2}}{\partial \lambda_2} = w(T_2 - L_2) + M - PX_2 = 0$

This gives:

(A19) $wu_X^{'} = Pu_L^{'}$

(A20) $w(T_2 - L_2) + M - PX_2 = 0$

Equations (A19) and (A20) determine X_2 and L_2.

The second order conditions can be written as follows:

(A21)

Appendix B: Total differentiation

1. Imperfect credit market – period 1

In period 1, the following system of equations based on (A8)-(A10) gives the optimal variables X_1^*, L_1^* and Z^* given exogenous values of E, $\overline{Z}$, T_1, w, P, a and c:

(B1) $wu'_X - Pu'_L = 0$

(B2) $(w(1+a)+c)u'_L - w\beta V'_N = 0$

(B3) $PX_1 + cZ - w(T_1 - L_1 - (1+a)Z) = 0$

A total differentiation of the system (B1)-(B3) gives:

(B4) $u'_X dw + w(u''_{XX}dX_1 + u''_{XL}dL_1 + u''_{XE}dE) - u'_L dP - P(u''_{LX}dX_1 + u''_{LL}dL_1 + u''_{LE}dE) = 0$

(B5) $$\begin{aligned}&u'_L(dw(1+a) + wda + dc) + (w(1+a)+c)(u''_{LX}dX_1 + u''_{LL}dL_1 + u''_{LE}dE)\\&-\beta V'_N dw - w\beta V''_{NN}(dZ + d\overline{Z}) = 0\end{aligned}$$

(B6) $$\begin{aligned}&X_1 dP + PdX_1 + cdZ + Zdc - (T_1 - L_1 - (1+a)Z)dw\\&-w(dT_1 - dL_1 - (1+a)dZ - Zda) = 0\end{aligned}$$

This can be written in the following way:

(B7) $u'_X dw - u'_L dP + (wu''_{XX} - Pu''_{LX})dX_1 + (wu''_{XL} - Pu''_{LL})dL_1 + (wu''_{XE} - Pu''_{LE})dE = 0$

(B8) $$\begin{aligned}&(u'_L(1+a) - \beta V'_N)dw + u'_L dc + (w(1+a)+c)u''_{LX}dX_1 + (w(1+a)+c)u''_{LL}dL_1\\&+(w(1+a)+c)u''_{LE}dE + u'_L wda - w\beta V''_{NN}dZ - w\beta V''_{NN}d\overline{Z} = 0\end{aligned}$$

(B9) $$\begin{aligned}&X_1 dP + PdX_1 + Zdc - (T_1 - L_1 - Z(1+a))dw - wdT_1 + wdL_1\\&+\left(w(1+a)+c\right)dZ + Zwda = 0\end{aligned}$$

<u>A. dE > 0, d$\overline{Z}$ = $dT_1 = dw = dP = dc = da = 0$</u>

(B7)-(B9) reduces to:

(B10) $(wu''_{XX} - Pu''_{LX})dX_1 + (wu''_{XL} - Pu''_{LL})dL_1 + (wu''_{XE} - Pu''_{LE})dE = 0$

(B11) $(w(1+a)+c)u''_{LX}dX_1 + (w(1+a)+c)u''_{LL}dL_1 + (w(1+a)+c)u''_{LE}dE - w\beta V''_{NN}dZ = 0$

(B12) $PdX_1 + wdL_1 + \left(w(1+a)+c\right)dZ = 0$

This gives:

(B13)
$$\frac{dX_1}{dE} = \frac{K_1}{\Delta}$$
$$K_1 = -(c+w(1+a))^2[u^{"}_{LE}u^{"}_{LX} - u^{"}_{LL}u^{"}_{XE}] + w\beta V^{"}_{NN}(wu^{"}_{XE} - Pu^{"}_{LE})$$

We know from the first order condition (B1) that $\mathrm{wU}^{'}_{\mathrm{X}} = \mathrm{PU}^{'}_{\mathrm{L}}$. Thus,

(B14) $\mathrm{wU}^{"}_{\mathrm{XE}} = \mathrm{PU}^{"}_{\mathrm{LE}}$. *Check this.*

Using this, we find:

(B15) $K_1 = -(c+w(1+a))^2[u^{"}_{LE}u^{"}_{LX} - u^{"}_{LL}u^{"}_{XE}] < 0$

The effect on L

(B16)
$$\frac{dL_1}{dE} = \frac{K_2}{\Delta}$$
$$K_2 = -(c+w(1+a))^2[u^{"}_{LX}u^{"}_{XE} - u^{"}_{LE}u^{"}_{XX}] + P\beta V^{"}_{NN}(Pu^{"}_{LE} - wu^{"}_{XE})$$

Using (B14), we find

(B17) $K_2 = -(c+w(1+a))^2[u^{"}_{LX}u^{"}_{XE} - u^{"}_{LE}u^{"}_{XX}] < 0$

Finally, the effect on Z is

$$\frac{dZ}{dE}=\frac{K_3}{\Delta}$$

(B18)

$$K_3=(c+w(1+a))[Pu^{"}_{LE}u^{"}_{LX}-Pu^{"}_{LL}u^{"}_{XE}+wu^{"}_{LX}u^{"}_{XE}-wu^{"}_{LE}u^{"}_{XX})>0$$

where

(B19) $\Delta=(c+w(1+a))^2[(u^{"}_{LX})^2-u^{"}_{LL}u^{"}_{XX}]-\beta V^{"}_{NN}(P^2u^{"}_{LL}-2Pwu^{"}_{LX}+w^2u^{"}_{XX})$

Note: Maybe we can use the second order conditions to find the sign of Δ? Can we show that Δ < 0?

B. dw > 0, $dE = d\overline{Z} = dT_1 = dP = dc = da = 0$

(B7)-(B9) reduces to:

(B20) $u^{'}_{X}dw+(wu^{"}_{XX}-Pu^{"}_{LX})dX_1+(wu^{"}_{XL}-Pu^{"}_{LL})dL_1=0$

(B21) $(u^{'}_{L}(1+a)-\beta V^{'}_{N})dw+(w(1+a)+c)u^{"}_{LX}dX_1+(w(1+a)+c)u^{"}_{LL}dL_1-w\beta V^{"}_{NN}dZ=0$

(B22) $PdX_1-(T_1-L_1-Z(1+a))dw+wdL_1+\left(w(1+a)+c\right)dZ=0$

This gives:

$$\frac{dX_1}{dw}=\frac{K_4}{\Delta}$$

(B23)
$$\begin{aligned}K_4=\frac{1}{w}&\left\{c^2U^{"}_{LL}U^{'}_{X}+(1+a)c\left(PU^{'}_{L}U^{"}_{LL}-wU^{'}_{L}U^{"}_{LX}+2wU^{"}_{LL}U^{'}_{X}\right)+c\beta V^{'}_{N}(wU^{"}_{LX}-PU^{"}_{LL})\right\}\\&+PU^{"}_{LL}\left((1+a)^2U^{'}_{L}+\beta[V^{"}_{NN}(Z(1+a)+L_1-T_1)-(1+a)V^{'}_{N}]\right)\\&+w\left((1+a)^2U^{"}_{LL}U^{'}_{X}-(1+a)^2U^{'}_{L}U^{"}_{LX}+\beta\left[U^{'}_{X}V^{"}_{NN}+U^{"}_{LX}\left((1+a)V^{'}_{N}-V^{"}_{NN}(Z(1+a)+L_1-T_1\right)\right]\right)\end{aligned}$$

$$\frac{dL_1}{dw}=\frac{-K_5}{\Delta}$$

(B24) $$K_5=\frac{1}{w}\left\{c^2U^{"}_{LX}U^{'}_{X}+(1+a)c\left(PU^{'}_{L}U^{"}_{LX}-wU^{'}_{L}U^{"}_{XX}+2wU^{"}_{LX}U^{'}_{X}\right)+c\beta V^{'}_{N}(wU^{"}_{XX}-PU^{"}_{LX})\right\}$$
$$+P\left((1+a)^2U^{'}_{L}U^{"}_{LX}+\beta\left[U^{'}_{X}V^{"}_{NN}+U^{"}_{LX}\left(V^{"}_{NN}(Z(1+a)+L_1-T_1)-(1+a)V^{'}_{N}\right)\right]\right)$$
$$+w\left((1+a)^2U^{"}_{LX}U^{'}_{X}-U^{"}_{XX}\left[(1+a)^2U^{'}_{L}+\beta\left[V^{"}_{NN}\left(Z(1+a)+L_1-T_1\right)-(1+a)V^{'}_{N}\right]\right]\right)$$

$$\frac{dZ}{dw}=\frac{-K_6}{\Delta}$$

(B25) $$K_6=\frac{1}{w}\left\{P^2U^{"}_{LL}\left((1+a)U^{'}_{L}-\beta V^{'}_{N}\right)+P\left(U^{'}_{X}U^{"}_{LL}\left(c+w(1+a)\right)-2w(1+a)U^{'}_{L}U^{"}_{LX}+2w\beta U^{"}_{LX}V^{'}_{N}\right)\right\}$$
$$+\left(w(1+a)+c\right)\left((U^{"}_{LX})^2\left(L_1+(1+a)Z-T_1\right)-U^{"}_{XX}U^{"}_{LL}(L_1+(1+a)Z-T_1)-U^{"}_{LX}U^{'}_{X}\right)$$
$$+w(1+a)U^{"}_{XX}U^{'}_{L}-w\beta V^{'}_{N}U^{"}_{XX}$$

K_4, K_5 and K_6 are indeterminate.

<u>C. d$\overline{Z}$ > 0, *dE = dT*$_1$ = dw = *dP = dc = da = 0*</u>

(B7)-(B9) reduces to:

(B26) $(wu^{"}_{XX}-Pu^{"}_{LX})dX_1+(wu^{"}_{XL}-Pu^{"}_{LL})dL_1=0$

(B27) $(w(1+a)+c)u^{"}_{LX}dX_1+(w(1+a)+c)u^{"}_{LL}dL_1-w\beta V^{"}_{NN}dZ-w\beta V^{"}_{NN}d\overline{Z}=0$

(B28) $PdX_1+wdL_1+\left(w(1+a)+c\right)dZ=0$

This gives

$$\frac{dX_1}{d\overline{Z}}=\frac{K_7}{\Delta}$$

(B29)

$$K_7=-\left((1+a)w+c\right)\beta V^{"}_{NN}(PU^{"}_{LL}-wU^{"}_{LX})<0$$

$$\frac{dL_1}{d\overline{Z}} = \frac{K_8}{\Delta}$$

(B30)

$$K_8 = -\left(w(1+a)+c\right)\beta V^{"}_{NN}(wU^{"}_{XX} - PU^{"}_{LX}) < 0$$

$$\frac{dZ}{d\overline{Z}} = \frac{K_9}{\Delta}$$

(B31)

$$K_9 = -\beta V^{"}_{NN}(P^2U^{"}_{LL} - 2wPU^{"}_{LX} + w^2U^{"}_{XX}) < 0$$

K_9 should be positive? Note that K_9 is equal to the last part of Δ and may be determined by second order conditions.

D. $dc > 0$, $dE = d\overline{Z} = dT_l = \mathrm{dw} = dP = da = 0$

(B7)-(B9) reduces to:

(B32) $(wu^{"}_{XX} - Pu^{"}_{LX})dX_1 + (wu^{"}_{XL} - Pu^{"}_{LL})dL_1 = 0$

(B33) $u^{'}_{L}dc + (w(1+a)+c)u^{"}_{LX}dX_1 + (w(1+a)+c)u^{"}_{LL}dL_1 - w\beta V^{"}_{NN}dZ = 0$

(B34) $PdX_1 + Zdc_1 + wdL_1 + \left(w(1+a)+c\right)dZ = 0$

This gives:

$$\frac{dX_1}{dc} = \frac{\mathrm{K}_{10}}{\Delta}$$

(B35)

$$K_{10} = (PU^{"}_{LL} - wU^{"}_{LX})\left(U^{'}_{L}(w(1+a)+c) + Zw\beta V^{"}_{NN}\right)$$

$$\frac{dL_1}{dc} = \frac{K_{11}}{\Delta}$$

(B36)

$$K_{11} = (wU^{"}_{XX} - PU^{"}_{LX})\left(U^{'}_{L}(w(1+a)+c) + Zw\beta V^{"}_{NN}\right)$$

(B37)

$$\frac{dZ}{dc} = \frac{K_{12}}{\Delta}$$

$$K_{12} = P^2 U_L^{'} U_{LL}^{"} - 2wPU_L^{'} U_{LX}^{"} + w\left(wU_L^{'} U_{XX}^{"} + \left((U_{LX}^{"})^2 - U_{LL}^{"} U_{XX}^{"}\right)(w(1+a)+c)Z\right)$$

K_{10}, K_{11} and K_{12} are indeterminate.

K_{12} should be positive? Note that $\left((U_{LX}^{"})^2 - U_{LL}^{"} U_{XX}^{"}\right)$ *is also found in Δ and may be determined by second order conditions.*

<u>E. *da* > 0, *dE* = d$\overline{Z}$ = *dT₁* = dw = *dP* = *dc* = *0*</u>

(B7)-(B9) reduces to:

(B38) $(wu_{XX}^{"} - Pu_{LX}^{"})dX_1 + (wu_{XL}^{"} - Pu_{LL}^{"})dL_1 = 0$

(B39) $(w(1+a)+c)u_{LX}^{"} dX_1 + (w(1+a)+c)u_{LL}^{"} dL_1 + u_L^{'} wda - w\beta V_{NN}^{"} dZ = 0$

(B40) $PdX_1 + wdL_1 + \left(w(1+a)+c\right)dZ + Zwda = 0$

This gives:

(B41)

$$\frac{dX_1}{da} = \frac{K_{13}}{\Delta}$$

$$K_{13} = -(PU_{LL}^{"} - wU_{LX}^{"})(\left(w(1+a)+c\right)U_L^{'} + w\beta V_{NN}^{"} Z)$$

(B42)

$$\frac{dL_1}{da} = \frac{K_{14}}{\Delta}$$

$$K_{14} = -(PU_{LX}^{"} - wU_{XX}^{"})(\left(w(1+a)+c\right)U_L^{'} + w\beta V_{NN}^{"} Z)$$

$$\frac{dZ}{da}=\frac{K_{15}}{\Delta}$$

(B43)

$$K_{15}=-(P^2U_L^{'}U_{LL}^{"}-2wPU_L^{'}U_{LX}^{"}+w\Big(wU_L^{'}U_{XX}^{"}+\Big((U_{LX}^{"})^2-U_{LL}^{"}U_{XX}^{"}\Big)\Big(w(1+a)+c\Big)Z\Big)$$

K_{13}, K_{14} and K_{15} are indeterminate.

K_{15} should be positive? Note that K_{15} =-K_{12}.

2. Imperfect credit market – period 2

In period 2, the equations (A19) and (A20) give the optimal variables X_2^*, L_2^* given exogenous values of E, T_2, w, P and M:

(B44) $wu_X^{'}-Pu_L^{'}=0$

(B45) $PX_2 - w(T_2 - L_2) - M = 0$

A total differentiation of the system (B44) and (B45) gives:

(B46) $u_X^{'}dw-u_L^{'}dP+(wu_{XX}^{"}-Pu_{LX}^{"})dX_2+(wu_{XL}^{"}-Pu_{LL}^{"})dL_2+(wu_{XE}^{"}-Pu_{LE}^{"})dE=0$

(B47) $X_2dP+PdX_2-(T_2-L_2)dw-wdT_2+wdL_2-dM=0$

<u>A. dE > 0, $dT_2 = dw = dP = dM = 0$</u>

(B46)-(B47) reduces to:

(B48) $(wu_{XX}^{"}-Pu_{LX}^{"})dX_2+(wu_{XL}^{"}-Pu_{LL}^{"})dL_2+(wu_{XE}^{"}-Pu_{LE}^{"})dE=0$

(B49) $PdX_2+wdL_2=0$

This gives

(B50) $\dfrac{dX_2}{dE}=\dfrac{w(Pu_{LE}^{"}-wu_{XE}^{"})}{\Lambda}$

(B51) $\dfrac{dL_2}{dE} = \dfrac{P(wu''_{XE} - Pu''_{LE})}{\Lambda}$

where

(B52) $\Lambda = P^2 u''_{LL} + w(wu''_{XX} - 2Pu''_{LX}) < 0$

The numerators in (B50) and (B51) are indeterminate.

<u>B. dw > 0, dE= *dT*$_2$ *= dP = dM = 0*</u>

(B46)-(B47) reduces to:

(B53) $u'_X dw + (wu''_{XX} - Pu''_{LX})dX_2 + (wu''_{XL} - Pu''_{LL})dL_2 = 0$

(B54) $PdX_2 - (T_2 - L_2)dw + wdL_2 = 0$

This gives

(B55) $\dfrac{dX_2}{dw} = \dfrac{-\left(wu'_X + (L_2 - T_2)(Pu''_{LL} - wu''_{LX})\right)}{\Lambda} > 0$

(B56) $\dfrac{dL_2}{dw} = \dfrac{Pu'_X + (L_2 - T_2)(Pu''_{LX} - wu''_{XX})}{\Lambda}$

The numerator in (B56) is indeterminate.

<u>C. dT$_2$ > 0, dE= *dw = dP = dM = 0*</u>

(B46)-(B47) reduces to:

(B57) $(wu''_{XX} - Pu''_{LX})dX_2 + (wu''_{XL} - Pu''_{LL})dL_2 = 0$

(B58) $PdX_2 - wdT_2 + wdL_2 = 0$

This gives

(B59) $\frac{dX_2}{dT_2} = \frac{w(Pu''_{LL} - wu''_{LX})}{\Lambda} > 0$

(B60) $\frac{dL_2}{dT_2} = \frac{w(wu''_{XX} - Pu''_{LX})}{\Lambda} > 0$

D. *dM* > 0, dE= *dw* = dT$_2$ = *dP* = *0*

(B46)-(B47) reduces to:

(B61) $(wu''_{XX} - Pu''_{LX})dX_2 + (wu''_{XL} - Pu''_{LL})dL_2 = 0$

(B62) $PdX_2 + wdL_2 - dM = 0$

This gives

(B63) $\frac{dX_2}{dM} = \frac{Pu''_{LL} - wu''_{LX}}{\Lambda} > 0$

(B64) $\frac{dL_2}{dM} = \frac{wu''_{XX} - Pu''_{LX}}{\Lambda} > 0$

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